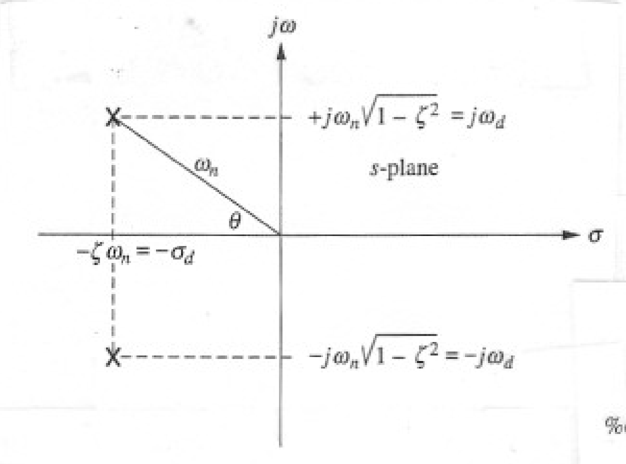
- In lecture 3 we saw how the parameters rise time (t_r), % overshoot, settling time (t_s) and the peak time (t_p) were defined for a second order system $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ where

$$\%\text{ overshoot} = 100\, e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \;;\quad t_s = \frac{4}{\zeta\omega_n} \;;\quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

- In many situations we can approximate the response of a higher order system with a generic second order system. In this manner we can use the root locus to determine the response of a more complicated system by knowing the general behavior of a second order system.

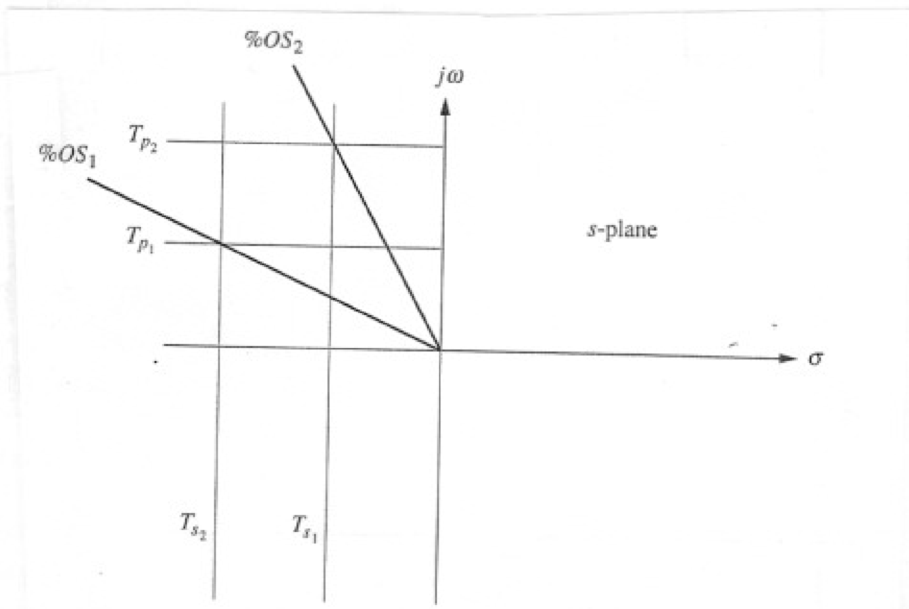


radial lines → ζ = constant

$\zeta = \cos\theta$

horizontal lines → t_p = constant

Vertical lines → t_s = constant

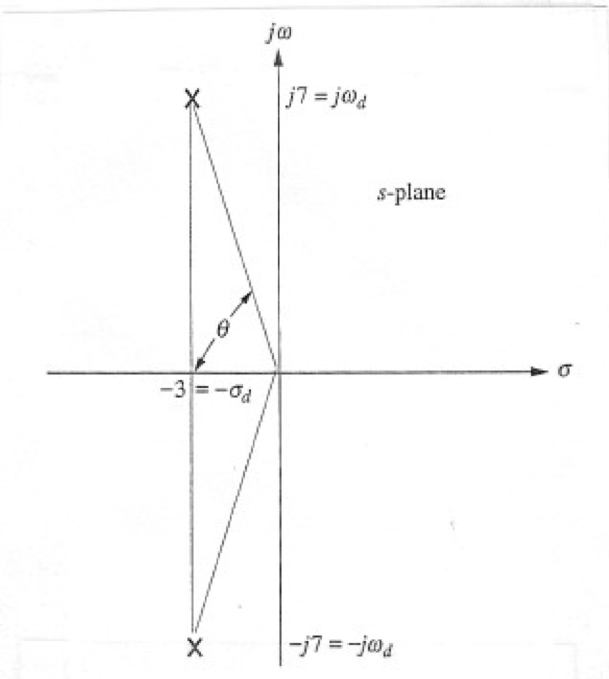


Lines of constant peak time, T_p, settling time, T_s, and percent overshoot, $\%OS$
Note: $T_{s_2} < T_{s_1}$; $T_{p_2} < T_{p_1}$; $\%OS_1 < \%OS_2$

The step response of the system as the poles moved was shown on page 4-1

Example Given the pole locations shown below

Find: ζ, ω_n, t_p, $\%OS$, t_s



$$\zeta = \cos\theta = \cos\left\{\tan^{-1}\left(\frac{7}{3}\right)\right\}$$

$$\zeta = 0.394$$

$$\omega_n = \sqrt{7^2 + 3^2} = 7.616 \text{ r/s}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{7} = 0.449 s$$

$$\%OS = 100\, e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}$$

$$\%OS = 26\%$$

$$t_s \approx \frac{4}{-\sigma_d} = \frac{4}{3} = 1.333 \text{ s}$$

- The equations for the parameters above are only valid for a system with two complex poles and no zeros. However under certain conditions a system with more than two poles or with zeros can be approximated by a second order system that has just two complex dominant poles. Once we justify this approximation, the above equations can be applied using the location of the dominant poles.

Let's consider the following plant that has a set of complex poles at $-\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$ and a real pole at $-\alpha_r$

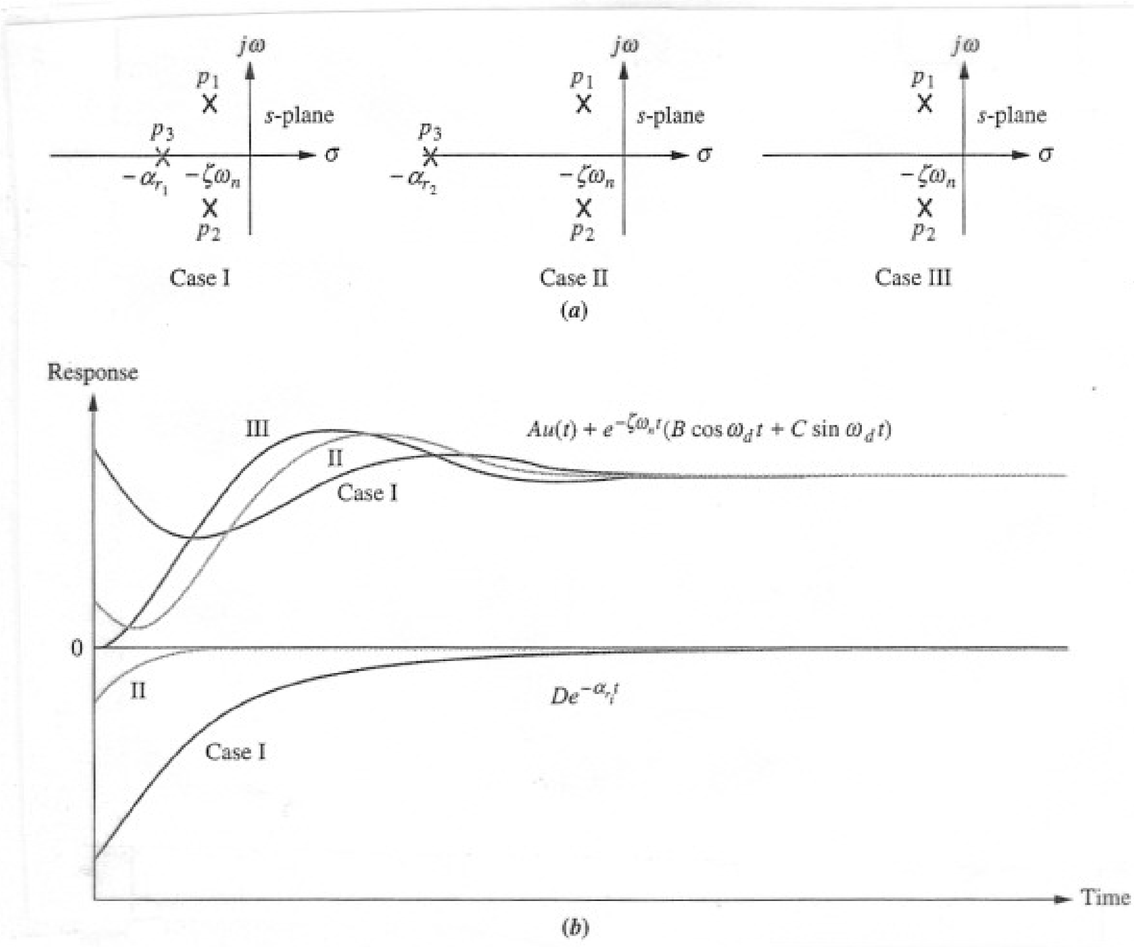
The step response of the system is given by:

$$C(s) = \frac{A}{s} + \frac{B(s+\zeta\omega_n) + C\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{D}{s+\alpha_r}$$

Taking the inverse Laplace transform of $C(s) \rightarrow$

$$c(t) = A u(t) + e^{-\zeta\omega_n t}\left(B\cos\omega_d t + C\sin\omega_d t\right) + De^{-\alpha_r t}$$

I) $\alpha_r \approx \zeta\omega_n$ II) $\alpha_r \gg \zeta\omega_n$ III) $\alpha_r = \infty$



Note: As the pole moves further to the left its contribution becomes smaller $\rightarrow$ the complex poles dominate!

<u>Rule of thumb</u>: you can assume that the exponential decay is negligible after five time constants. Therefore if the real pole is five times further to the left than the dominant poles we can assume that the system can be represented by its dominant second-order pair of poles.

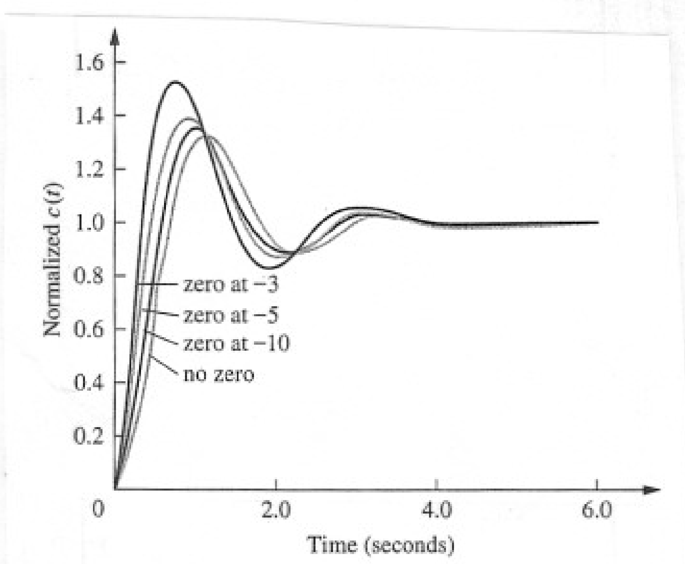
Addition of Zeros

Assume we have a two pole system and a single zero

$$C(s) = \frac{(s+a)}{(s+1+i\,2.828)(s+1-i\,2.828)}$$

Zero located at $-a$

The step response of the system is shown below for zeros at $-3, -5, -10$ and no zero



The closer the zero is to the dominant pole, the greater the effect on the transient response

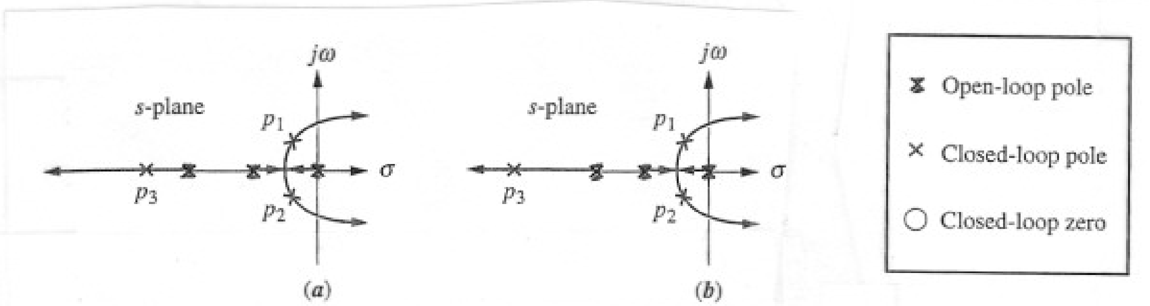
- Let's consider a plant $G(s)$ with unity in the numerator. If we add a zero to the transfer function, the new transfer function will be

$$G_{new}(s) = (s+a)\,G(s) = s\,G(s) + a\,G(s)$$

The response of the new system consists of the derivative of the original response plus a scaled version of the original response. If a is very large, then the derivative component is not significant. As a becomes smaller the derivative term contributes more to the response and has a greater effect. For a 2nd order system, if a is small, we can expect more overshoot.

Summary of conditions justifying a second order approximation

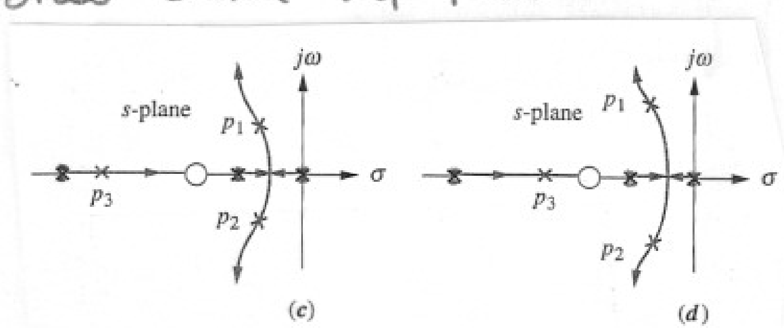
① Higher order poles are much farther into the left half s-plane than the dominant second order pair of poles



(a) (b)

(b) would yield a better second order approximation since the closed loop pole p_3 is further to the left

② Closed loop zeros near the closed loop second order pole pair are nearly canceled by the proximity of higher order closed loop poles



(c) (d)

(d) yields a better approximation because the pole p_3 is closer to canceling the closed loop zero

(3) Closed loop zeros not canceled by the close proximity of higher order closed loop poles are far removed from the closed loop second order pole pair

Design Procedure for Higher Order Systems

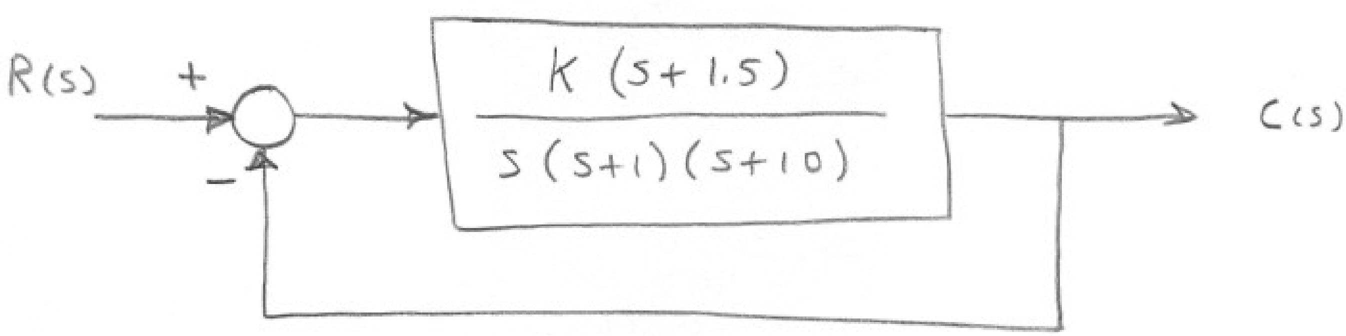
(1) sketch the root locus

(2) Assume the system is a second order system w/out any zeros and then find the gain to meet the transient response specification

(3) Justify your second order assumption

(4) If the assumptions can not be justified, your simulation will have to be simulated in order to be sure it meets the transient response specification. It is a good idea to simulate the system anyway.

<u>Example</u> Design the value of gain K to yield a 1.52% overshoot. Is the second order assumption valid?



The root locus is plotted using the "rlocus" command
the gain K is found by using the "rlocfind" command